

Test of the Glauber Formula for Nucleon-Deuteron Scattering at Intermediate Energies

V. I. Kovalchuk

Department of Physics, Taras Shevchenko National University, Kiev 01033, Ukraine

Abstract

A high-precision test of the Glauber formula for the amplitude of nucleon-deuteron scattering is performed. Nucleon-nucleon amplitudes used in the calculations depend on the spins of interacting particles, phase shifts, and mixing parameters. These amplitudes were derived by using the Nijmegen potentials. The differential cross sections for nucleon-deuteron scattering were calculated for the projectile-nucleon energies of 65, 95, 135, 150, 190, and 250 MeV, and the results of these calculations were compared with experimental data.

PACS numbers: 03.65.Nk, 21.45.-v, 24.10.Ht, 25.10.+s

1. Introduction

Investigation of collisions between high-energy particles and nuclei is an important problem within which one can study nucleon-density distributions, the dynamics of nucleon-cluster formation in nuclei, the color properties of quark structures, and so on. From the microscopic point of view, a consistent description of quantities observed in these reactions is an extremely difficult theoretical problem [1]. In view of this, the number of currently existing successful theoretical approaches is relatively small here, the Glauber method being among them [2, 3]. In [1], it was indicated, with good reason, that, although this important method has extensively been used in various scattering problems for more than half a century, only very few studies, surprisingly as it is, have been devoted to rigorously verifying the accuracy and the applicability range of the Glauber ansatz. It is the opinion of the present author that, for a first step along these lines, we could address the problem of a more rigorous derivation of form for the nucleon-nucleon amplitude in elastic nucleon-deuteron scattering, which is the simplest nucleon-nucleus reaction. The amplitude obtained in [4] from the conditions of invariance of the nucleon-nucleon interaction with respect to spatial rotations and space and time inversions could be such a form

of representation of the nucleon-nucleon amplitude. This amplitude has the most general form and depends on the spins of colliding particles, their energy, phase shifts, and mixing parameters. Using it in the well-known formula of the eikonal approximation for the amplitude of nucleon scattering on a deuteron [3], we can directly test the correctness of the Glauber approach for the case being considered.

2. Formalism

The differential cross section for nucleon-deuteron scattering is calculated here by the formula [5] (below, use is everywhere made of the c. m. frame and the system of units where $\hbar=c=1$)

$$\sigma(\theta) \equiv \frac{d\sigma}{d\Omega} = \frac{1}{6} \text{Tr}(F_d F_d^\dagger), \quad (1)$$

where F_d is the amplitude for nucleon-deuteron scattering. It has the form [3]

$$F_d(\mathbf{q}) = k_d(k_1^{-1} F_1(\mathbf{q}) + k_2^{-1} F_2(\mathbf{q})) G(\mathbf{q}/2) + \frac{ik_d}{2\pi k_1 k_2} \int d^{(2)}\mathbf{g} F_1(\mathbf{g} + \mathbf{q}/2) F_2(\mathbf{g} - \mathbf{q}/2) G(\mathbf{g}), \quad (2)$$

$$G(\mathbf{g}) = \int d^{(3)}\mathbf{r} |\phi_d(\mathbf{r})|^2 \exp(i\mathbf{g}\mathbf{r}), \quad \mathbf{q}, \mathbf{g} \perp \mathbf{k}_d, \quad (3)$$

where \mathbf{k}_d is the deuteron momentum, k_1 and k_2 are the momenta of the deuteron nucleons, $F_{1,2}$ are the amplitudes for projectile-nucleon scattering on the deuteron nucleons, \mathbf{q} is the momentum transfer, and $\phi_d(\mathbf{r})$ is the ground-state deuteron wave function.

The sum of the first two terms in expression (2) is the scattering amplitude in the impulse approximation, $F_d^{(i)}$ (it corresponds to taking into account single collisions between the projectile nucleon and the deuteron nucleons). The last term in expression (2) is the so-called shadowing correction $F_d^{(sh)}$; it corresponds to the contribution of double scattering to the amplitude, F_d :

$$F_d = F_d^{(i)} + F_d^{(sh)}. \quad (4)$$

With allowance for the charge invariance of the nucleon-nucleon interaction, the amplitudes F_j ($j = 1, 2$) have the form [4, 6]

$$F_j = f_{1j} + f_{2j}(\mathbf{n}\boldsymbol{\sigma}_j)(\mathbf{n}\boldsymbol{\sigma}) + if_{3j}\mathbf{n}(\boldsymbol{\sigma}_j + \boldsymbol{\sigma}) + f_{4j}(\mathbf{m}\boldsymbol{\sigma}_j)(\mathbf{m}\boldsymbol{\sigma}) + f_{5j}(\mathbf{l}\boldsymbol{\sigma}_j)(\mathbf{l}\boldsymbol{\sigma}), \quad (5)$$

where f_{sj} ($s = 1, 2, \dots, 5$) are coefficients that depend on the energy of colliding nucleons, phase shifts, and mixing parameters (see Appendix A); $\boldsymbol{\sigma}$ is the projectile-nucleon spin operator; and \mathbf{n} , \mathbf{m} , and \mathbf{l} are three mutually orthogonal unit vectors defined as

$$\mathbf{n} = \frac{\mathbf{k}_j \times \mathbf{k}'_j}{|\mathbf{k}_j \times \mathbf{k}'_j|}, \quad \mathbf{m} = \frac{\mathbf{k}_j - \mathbf{k}'_j}{|\mathbf{k}_j - \mathbf{k}'_j|}, \quad \mathbf{l} = \frac{\mathbf{k}_j + \mathbf{k}'_j}{|\mathbf{k}_j + \mathbf{k}'_j|},$$

where \mathbf{k}_j (\mathbf{k}'_j) is the momentum of the incident (scattered) j th nucleon ($k_j = k_d/2 = k$).

Evaluation of the trace in Eq. (1) with allowance for Eqs. (2) and (5) leads to the expression

$$Tr(f_d f_d^\dagger) = 4|G(\mathbf{q}/2)|^2 S_{11} + 2(\pi k)^{-1} G(\mathbf{q}/2) S_{12} + (\pi k)^{-2} S_{22}, \quad (6)$$

where S_{11} , S_{12} , and S_{22} are quantities that depend on the coefficients appearing in the amplitudes f_j (see Appendix B). The calculations of the differential cross sections in (1) were performed for the projectile-nucleon energies of 65, 95, 135, 150, 190, and 250 MeV by using the nucleon-nucleon phase shifts, mixing parameters, and deuteron wave functions obtained with the Nijm I, Nijm II, Nijm 93, and Reid 93 potentials [7]. Here, we disregarded relativistic corrections, since, in the energy range being considered, their effect on the cross section is insignificant (see [1] and references therein).

3. Analysis of calculation results. Conclusions

An analysis of the calculated cross sections in the figure leads to the following conclusions:

(i) For a given projectile-nucleon energy E_N , the relative contribution of double scattering, $F_d^{(sh)}$, is virtually independent of the angle θ and decreases as E_N grows. In the Glauber approximation (the deuteron size exceeds considerably the range of the nucleon-nucleon interaction), the shadowing correction decreases more slowly in relation to $F_d^{(i)}$ the angle θ grows [18, 19].

(ii) It is traditionally assumed [2, 20] that the Glauber formula (2) is valid for $q \ll k$ or $k \ll R_{rms}$, where R_{rms} is the root-mean-square radius of the deuteron. This condition is independent of energy and yields the following estimate for the angle θ : $\theta < 10^\circ$. In fact, the results of precise calculations performed in the present study for the aforementioned cross sections with the above realistic potentials show that the Glauber formalism works well even beyond the original assumptions of the

theory: the calculated curves describe experimental data in the angular range $\theta < 30^\circ$ and in the energy range $65 \div 150$ MeV; as the projectile-nucleon energy increases further, the angular range becomes broader: $\theta < 70^\circ$ (for 190 and 250 MeV).

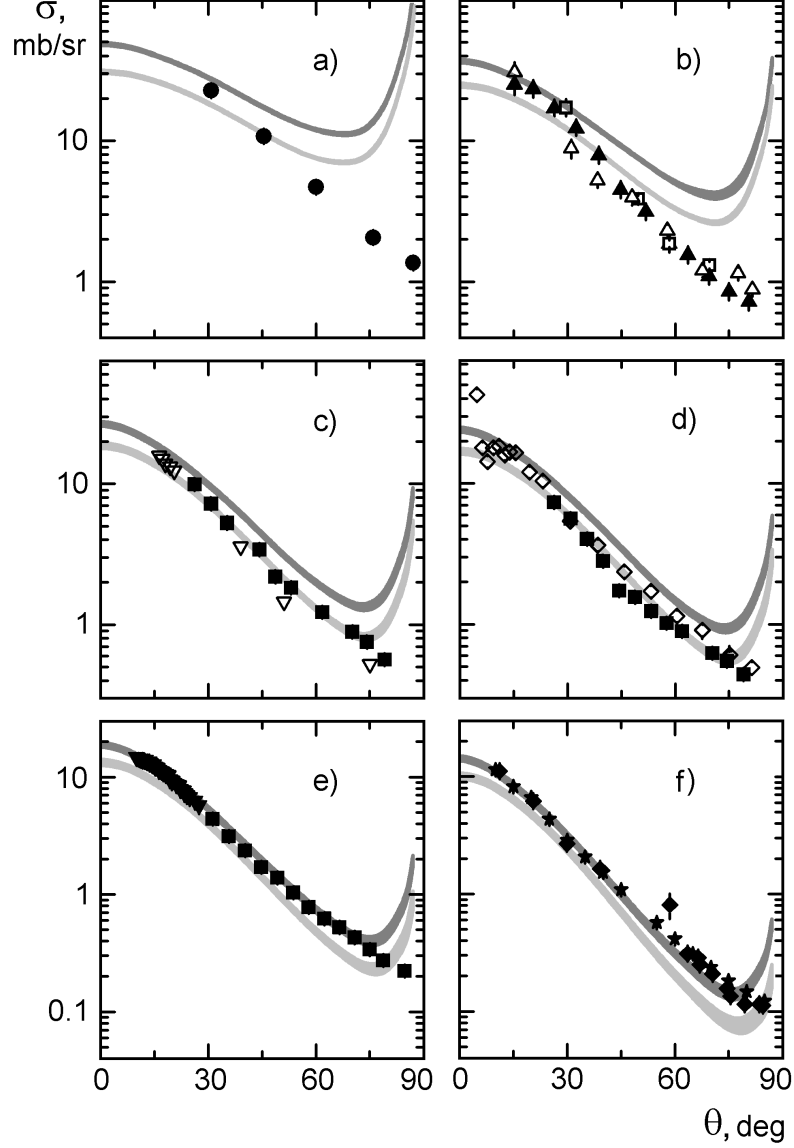


Figure 1: Differential cross sections for nucleon-deuteron scattering according to the calculations with the Nijm I, Nijm II, Nijm 93, and Reid 93 potentials for the energies of (a) 65, (b) 95, (c) 135, (d) 150, (e) 190, and (f) 250 MeV. The cross-section curves lie within the dark-gray bands. The light-gray bands correspond to the cross sections calculated in the impulse approximation. The displayed experimental data were borrowed from (\bullet) [8], (\blacktriangle) [9], (\triangle) [10], (\square) [11], (\blacksquare) [12], (∇) [13], (\diamond) [14], (\blacktriangledown) [15], (\blacklozenge) [16], and (\star) [17].

Thus, the test performed here for the Glauber formula in the case of nucleon-deuteron scattering by using realistic nucleon-nucleon potentials gives sufficient ground to conclude that, in the range of projectile-nucleon energies E_N that is considered here, the theory in question works well beyond the constraint $q \ll k$. Moreover, the angular range broadens to $\theta < 70^\circ$ as soon as the quantity k^{-1} becomes commensurate with R_{rms} , not only much less than it.

Appendix A

The coefficients f_s in the amplitude for nucleon-nucleon scattering in (5) are expressed in terms of the phase shifts $^1\delta_\ell^J$ and $^3\delta_\ell^J$ and the mixing parameters ϵ^J as (the index j on f_s is suppressed for the sake of convenience) [4, 5]

$$f_1 = \frac{1}{4k} \sum_{\ell=0}^{\infty} [(\ell+2)a_\ell^{\ell+1} + (2\ell+1)a_\ell^\ell + (\ell-1)a_\ell^{\ell-1} + (\ell+1)b_\ell^{\ell+1} + \ell b_\ell^{\ell-1} + (2\ell+1)c_\ell^\ell] P_\ell(\cos \theta); \quad (\text{A.1})$$

$$f_2 = \frac{1}{4k} \left\{ \sum_{\ell=0}^{\infty} [(\ell+1)b_\ell^{\ell+1} + \ell b_\ell^{\ell-1} - (2\ell+1)c_\ell^\ell] P_\ell(\cos \theta) - \sum_{\ell=2}^{\infty} \left[\frac{1}{\ell+1} a_\ell^{\ell+1} - \frac{2\ell+1}{\ell(\ell+1)} a_\ell^\ell + \frac{1}{\ell} a_{\ell-1}^\ell \right] P_\ell^2(\cos \theta) \right\}; \quad (\text{A.2})$$

$$f_3 = \frac{1}{4k} \sum_{\ell=1}^{\infty} \left[\frac{\ell+2}{\ell+1} a_\ell^{\ell+1} - \frac{2\ell+1}{\ell(\ell+1)} a_\ell^\ell - \frac{\ell-1}{\ell} a_\ell^{\ell-1} + b_\ell^{\ell+1} - b_\ell^{\ell-1} \right] P_\ell^1(\cos \theta); \quad (\text{A.3})$$

$$f_4 = \frac{1}{4k \cos \theta} \left\{ \sum_{\ell=0}^{\infty} \left[\frac{1}{2} \{ (\ell+2)a_\ell^{\ell+1} + (2\ell+1)a_\ell^\ell + (\ell-1)a_\ell^{\ell-1} \} (\cos \theta - 1) + (\ell+1)b_\ell^{\ell+1} + \ell b_\ell^{\ell-1} - (2\ell+1)c_\ell^\ell \cos \theta \right] P_\ell(\cos \theta) + \frac{1}{2} \sum_{\ell=2}^{\infty} \left[\frac{1}{\ell+1} a_\ell^{\ell+1} - \frac{2\ell+1}{\ell(\ell+1)} a_\ell^\ell + \frac{1}{\ell} a_{\ell-1}^\ell \right] (1 + \cos \theta) P_\ell^2(\cos \theta) \right\}; \quad (\text{A.4})$$

$$f_5 = \frac{1}{4k \cos \theta} \left\{ \sum_{\ell=0}^{\infty} \left[\frac{1}{2} \{ (\ell+2)a_\ell^{\ell+1} + (2\ell+1)a_\ell^\ell + (\ell-1)a_\ell^{\ell-1} \} (1 + \cos \theta) - \right. \right.$$

$$\begin{aligned}
& -(\ell+1)b_\ell^{\ell+1} - \ell b_\ell^{\ell-1} - (2\ell+1)c_\ell^\ell \cos \theta \Big] P_\ell(\cos \theta) + \\
& + \frac{1}{2} \sum_{\ell=2}^{\infty} \left[\frac{1}{\ell+1} a_\ell^{\ell+1} - \frac{2\ell+1}{\ell(\ell+1)} a_\ell^\ell + \frac{1}{\ell} a_\ell^{\ell-1} \right] (\cos \theta - 1) P_\ell^2(\cos \theta) \Big\}. \quad (\text{A.5})
\end{aligned}$$

In expressions (A.1)-(A.5), we have introduced the following notation:

$$\begin{aligned}
a_\ell^{J=\ell} & \equiv \sin^3 \delta_J^J \exp(i^3 \delta_J^J), \\
a_\ell^{J=\ell+1} & \equiv \alpha^J \cos^2 \epsilon^J + \beta^J \sin^2 \epsilon^J + \frac{1}{2} \sqrt{\frac{J}{J+1}} (\alpha^J - \beta^J) \sin 2\epsilon^J, \\
b_\ell^{J=\ell+1} & \equiv \alpha^J \cos^2 \epsilon^J + \beta^J \sin^2 \epsilon^J - \frac{1}{2} \sqrt{\frac{J+1}{J}} (\alpha^J - \beta^J) \sin 2\epsilon^J, \\
a_\ell^{J=\ell-1} & \equiv \alpha^J \sin^2 \epsilon^J + \beta^J \cos^2 \epsilon^J + \frac{1}{2} \sqrt{\frac{J+1}{J}} (\alpha^J - \beta^J) \sin 2\epsilon^J, \\
b_\ell^{J=\ell-1} & \equiv \alpha^J \sin^2 \epsilon^J + \beta^J \cos^2 \epsilon^J - \frac{1}{2} \sqrt{\frac{J}{J+1}} (\alpha^J - \beta^J) \sin 2\epsilon^J, \\
\alpha^J & \equiv \sin^3 \delta_{J-1}^J \exp(i^3 \delta_{J-1}^J), \\
\beta^J & \equiv \sin^3 \delta_{J+1}^J \exp(i^3 \delta_{J+1}^J), \\
c_\ell^{J=\ell} & \equiv \sin^1 \delta_J^J \exp(i^1 \delta_J^J).
\end{aligned}$$

Appendix B

The quantities S_{11} , S_{12} , and S_{22} in (6) have the form

$$S_{11} = 4 \left\{ \Re(\alpha_1 \bar{\alpha}_2 + \gamma_1 \bar{\gamma}_2) + \sum_{j=1}^2 (|\alpha_j|^2/2 + |\beta_j|^2 + |\gamma_j|^2 + |\delta_j|^2 + |\epsilon_j|^2) \right\}; \quad (\text{B.1})$$

$$\begin{aligned}
S_{12} & = 8\Im(\bar{\beta}_1 a_{21} + \bar{\gamma}_1 a_{31} + \bar{\delta}_1 a_{41} + \bar{\epsilon}_1 a_{51}) + \\
& + 8\Im(\bar{\beta}_2 a_{12} + \bar{\gamma}_2 a_{13} + \bar{\delta}_2 a_{14} + \bar{\epsilon}_2 a_{15}) + \\
& + 4\Im\{(\bar{\alpha}_1 + \bar{\alpha}_2)(a_{11} + a_{33})\}; \quad (\text{B.2})
\end{aligned}$$

$$\begin{aligned}
S_{22} & = 2|a_{11}|^2 + 4|a_{12}|^2 + 4|a_{13}|^2 + 4|a_{14}|^2 + 4|a_{15}|^2 + \\
& + 4|a_{21}|^2 + 8|a_{22}|^2 + 12|a_{23}|^2 + 16|a_{24}|^2 + 16|a_{25}|^2 + \\
& + 4|a_{31}|^2 + 12|a_{32}|^2 + 14|a_{33}|^2 + 16|a_{34}|^2 + 16|a_{35}|^2 + \\
& + 4|a_{41}|^2 + 16|a_{42}|^2 + 16|a_{43}|^2 + 8|a_{44}|^2 + 16|a_{45}|^2 + \\
& + 4|a_{51}|^2 + 16|a_{52}|^2 + 16|a_{53}|^2 + 16|a_{54}|^2 + 8|a_{55}|^2 +
\end{aligned}$$

$$\begin{aligned}
& +4\Re[(\bar{a}_{13} + 2\bar{a}_{23})(a_{31} + 2a_{32}) - \\
& - 4\Re[\bar{a}_{33}(a_{11} + 2(a_{12} + a_{21}) + 4a_{22})],
\end{aligned} \tag{B.3}$$

where a_{ij} ($i, j = 1, 2, \dots, 5$) are double integrals of the form

$$a_{ij} = \int d^{(2)}\mathbf{g} G(\mathbf{g}) f_{i1}(\mathbf{g} + \mathbf{q}/2) f_{j2}(\mathbf{g} - \mathbf{q}/2). \tag{B.4}$$

References

1. Ch. Elster, T. Lin, W. Glöckle, and S. Jeschonnek, Phys. Rev. C **78**, 034002 (2008).
2. R. J. Glauber, Phys. Rev. **100**, 242 (1955).
3. V. Franco and R. J. Glauber, Phys. Rev. **142**, 1195 (1966).
4. L. Wolfenstein, Ann. Rev. Nucl. Sci. **6**, 43 (1956).
5. R. J. Seyler, Nucl. Phys. A **124**, 253 (1969).
6. M. L. Goldberger, Y. Nambu and R. Oehme, Ann. Phys. **2**, 226 (1957).
7. V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994); J. J. de Swart, R. A. M. Klomp, M. C. M. Rentmeester, and Th. A. Rijken, Few-Body Systems Suppl. **99**, 1 (2008); <http://nn-online.org>.
8. J. Balewski, K. Bodek, L. Jarczyk *et al.*, Nucl. Phys. A **581**, 131 (1995).
9. P. Mermod, J. Blomgren, A. Hildebrand *et al.*, Phys. Rev. C **72**, 061002 (2005).
10. P. Mermod, J. Blomgren, B. Bergenwall *et al.*, Phys. Lett. B **597**, 243 (2004).
11. O. Chamberlain and M. O. Stern, Phys. Rev. **94**, 666 (1954).
12. K. Ermisch, H. R. Armir-Ahmadi, A. M. van den Berg *et al.*, Phys. Rev. C **68**, 051001 (2003).
13. K. Sekiguchi, H. Sakai, H. Witała *et al.*, Phys. Rev. Lett. **95**, 162301 (2005).
14. H. Postma and R. Wilson, Phys. Rev. **121**, 1229 (1961).
15. H. Rohdjeß, W. Scobel, H. O. Meyer *et al.*, Phys. Rev. C **57**, 2111 (1998).
16. K. Hatanaka, Y. Shimizu, D. Hirooka *et al.*, Phys. Rev. C **66**, 044002 (2002).
17. Y. Maeda, H. Sakai, K. Fujita *et al.*, Phys. Rev. C **76**, 014004 (2007).

18. A. G. Sitenko, Sov. Phys. JETP **36**, 1008 (1959).
19. A. G. Sitenko, *Theory of Nuclear Reactions*, (World Scientific, Singapore, 1990).
20. D. R. Harrington, Phys. Rev. **135**, B358 (1964).